String-like structures in the four-dimensional Kerr geometry: Complexification as alternative to compactification

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Abstract

The 4d Kerr geometry displays many wonderful relations with quantum world and, in particular, with superstring theory. The lightlike structure of fields near the Kerr singular ring is similar to the structure of Sen solution for a closed heterotic string. Another string, open and complex, appears in the initiated by Newman complex representation of the Kerr geometry. Combination of these strings forms a membrane source of the Kerr geometry which is parallel to the string/M-theory unification. In this paper we give one more evidence of this relationship, emergence of the Calabi-Yau twofold (K3 surface) in twistorial structure of the Kerr geometry as a consequence of the Kerr theorem. Finally, we indicate that the Kerr stringy system may correspond to a complex embedding of the critical N=2 superstring.

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1 Introduction

The black hole solutions of diverse dimensions, represent now one of the basic objects for study in superstring theory. Recent ideas and methods in the black hole physics are based on complex analyticity and conformal field theory, which unifies the black hole physics with superstring theory and physics of elementary particles. The Kerr solution plays in these respect especial role. Being obtained as a metric of a "spinning mass" [1] with angular momentum $J = m|a|$, the Kerr solution found basic application as a metric of rotating black hole. In the four-dimensional Kerr solution, parameter $a = J/m$ is radius of the Kerr singular ring. For $|a| < m$, the ring is covered by horizon, but for parameters of the elementary particles $|a| >> m$, the black hole horizons disappear, and the Kerr singular ring turns out to be naked. Following the censorship principle, it should be covered by a source. During four decades of investigations, structure of Kerr's source was specified step for step. One of the earlier models was the model of the Kerr ring as a closed string [2, 3]. It has been obtained in [4] that structure of the fields around the Kerr ring is similar to the structure of the heterotic string in the obtained by Sen solutions to low energy string theory [5]. However, the Kerr string is branch line of the Kerr space-time into two sheets [6], and this bizarre peculiarity created an alternative line of investigations of the problem of Kerr's source,[7, 8, 9, 10, 11], which led to conclusion that the source of the Kerr-Newman (KN) solution should form a rigidly rotating membrane, or to be more precise, a highly oblate ellipsoidal bubble with a flat vacuum interior [9, 11].

The charged KN solution [12] has found application as a consistent with gravity classical model of spinning particle, [13, 14, 2, 8, 9, 15], which has gyromagnetic ratio $g = 2$, as that of the Dirac electron [13, 14], and also displays other relationships with the Dirac electron,[1] [16, 17, 18, 19], as well as the relationships with twistor theory [6, 20, 21], models of the soliton [10, 11, 22] and with basic structures of superstring theory [18, 21, 3, 4, 23].

In this note we consider complex structure of the Kerr geometry [23] and reveal one new evidence of its inherent parallelism with twistor theory and superstring theory. Namely, we show the presence of *the Calabi-Yau twofold* (K3 surface) in complex structure of the Kerr geometry, which appears as a consequence of the Kerr theorem in the form of a quartic equation in the projective twistor space CP^3. In section 2. we describe briefly the real structure of the Kerr geometry and the Kerr theorem, which determines Kerr's principal null congruence (PNC) in twistor terms.

On the way to our principal result, there appears a few important intermediate structures. First of all it is the complex Kerr geometry itself, which is generated by the Appel complex shift method, [24], and by the Newman complex retarded-time construction [25]. We describe them in section 3.

[1] In fact, the four observable parameters of the electron: mass, spin, charge and magnetic moment indicate unambiguously that the KN solution is to be the electron background geometry [18].

In section 4. we show that the source of the complex Kerr geometry is an open complex string. It is based on the old remarks by Ooguri and Vafa, that the complex world lines (CWL) parametrized by complex time parameter $\tau = t + i\sigma$, turns into a world-sheet of a complex string, [26], and the complex Kerr string generating the complex Kerr geometry, is to be an open string,[23] with orientifold world-sheet.

Finally, we observe that the structure of the membrane (vacuum bubble [11]) source of the *real* Kerr geometry is parallel to formation of the membrane in the superstring/M-theory unification: the closed Kerr's string of the real Kerr geometry grows by extra world-sheet parameter from the open complex Kerr string, [27], and formation of the Kerr bubble source is parallel to the model of enhancon [28].

The parallelism of the complex Kerr geometry with the basic structures of the superstring theory, and in particular, the inherent existence of the K3 surface in twistorial CP^3 space of the principal null congruences, allows us to suppose that the complex Kerr string represents a complex realization of the critical N=2 superstring theory, which may be embedded into complex Kerr geometry. We arrive at the point of view that complexification may represents an alternative to compactification.

2 Real structure of the KN geometry

KN metric is represented in the Kerr-Schild (KS) form [14],

$$g_{\mu\nu} = \eta_{\mu\nu} + 2he^3_\mu e^3_\nu, \tag{1}$$

where $\eta_{\mu\nu}$ is auxiliary Minkowski background in Cartesian coordinates $\mathrm{x} = x^\mu = (t, x, y, z)$,

$$h = P^2 \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}, \quad P = (1 + Y\bar{Y})/\sqrt{2}, \tag{2}$$

and $e^3(\mathrm{x})$ is a tangent direction to a *Principal Null Congruence (PNC)*, which is determined by the form[2]

$$e^3_\mu dx^\mu = du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv, \tag{3}$$

via function $Y(\mathrm{x})$, which is obtained from *the Kerr theorem*, [14, 29, 30, 31, 32, 33].

The PNC forms a caustic at the Kerr singular ring, $r = \cos\theta = 0$. As a result, the aligned with Kerr PNC metric and the KN electromagnetic potential,

$$A_\mu = -P^{-2} Re \frac{e}{(r + ia\cos\theta)} e^3_\mu, \tag{4}$$

[2]Here $\zeta = (x + iy)/\sqrt{2}, \quad \bar{\zeta} = (x - iy)/\sqrt{2}, u = (z + t)/\sqrt{2}, \quad v = (z - t)/\sqrt{2}$, are the null Cartesian coordinates, r, θ, ϕ are the Kerr oblate spheroidal coordinates, and $Y(\mathrm{x}) = \mathrm{e}^{\mathrm{i}\phi} \tan\frac{\theta}{2}$ is a projective angular coordinate. The used signature is $(-+++)$.

concentrate near the Kerr ring, forming a closed string – waveguide for traveling electromagnetic waves [2, 3, 18]. Analysis of the Kerr-Sen solution to low energy string theory [5] showed that similarity of the Kerr ring with a closed strings is not only analogue, but it has really the structure of a fundamental heterotic string [4]. Along with this closed string, the KN geometry contains also a *complex open string*, [23], which appears in the initiated by Newman complex representation of Kerr geometry, [25]. This string gives an extra dimension θ to the stringy source ($\theta \in [0, \pi]$), resulting in its extension to a membrane (bubble source [9, 11]. A superstring counterpart of this extension is a transfer from superstring theory to 11-dimensional M-theory and $M2$-brane, [27].

Kerr Theorem determines the shear free null congruences with tangent direction (3) by means of the solution $Y(\mathrm{x})$ of the equation

$$F(T^A) = 0, \tag{5}$$

where $F(T^A)$ is an arbitrary holomorphic function in the projective twistor space

$$T^A = \{Y, \quad \lambda^1 = \zeta - Yv, \quad \lambda^2 = u + Y\bar{\zeta}\}. \tag{6}$$

Using the Cartesian coordinates x^μ, one can rearrange variables and reduce function $F(T^A)$ to the form $F(Y, x^\mu)$, which allows one to get solution of the equation (5) in the form $Y(x^\mu)$.

For the Kerr and KN solutions, the function $F(Y, x^\mu)$ turns out to be quadratic in Y,

$$F = A(x^\mu)Y^2 + B(x^\mu)Y + C(x^\mu), \tag{7}$$

and the equation (5) represents a *quadric* in the projective twistor space $\mathbf{CP^3}$, with a non-degenerate determinant $\Delta = (B^2 - 4AC)^{1/2}$ which determines the complex radial distance [32, 34]

$$\tilde{r} = -\Delta = -(B^2 - 4AC)^{1/2}. \tag{8}$$

This case is explicitly resolved and yields two solutions

$$Y^{\pm}(x^\mu) = (-B \mp \tilde{r})/2A, \tag{9}$$

which allows one to restore two PNC by means of (3).
One can easily obtain from (7) and (9) that complex radial distance $\tilde{r}$ may also be determined from the relation

$$\tilde{r} = -dF/dY, \tag{10}$$

and therefore, the Kerr singular ring, $\tilde{r} = 0$, corresponds to caustics of the Kerr congruence,

$$dF/dY = 0. \tag{11}$$

As a consequence of the Vieta's formulas, the quadratic in Y function (7) may be expressed via the solutions $Y^{\pm}(x^\mu)$ in the form

$$F(Y, x^\mu) = A(Y - Y^+(x^\mu))(Y - Y^-(x^\mu)). \tag{12}$$

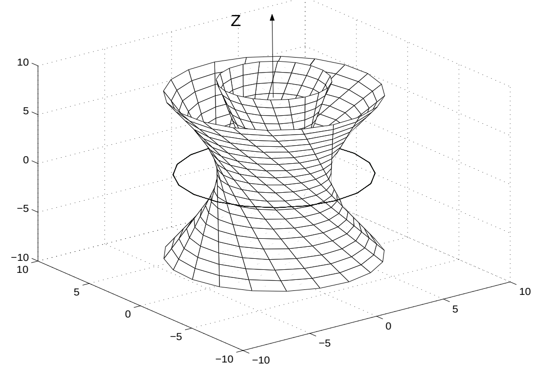

Figure 1: Twistor null lines of the Kerr congruence are focused on the Kerr singular ring, forming a twosheeted spacetime branched by closed string.

3 Complex Kerr geometry and the complex retarded-time construction

KN solution was initially obtained by a "complex trick" [12], and Newman [25] showed that linearized KN solution may be generated by a complex world line. This complex trick was first described by Appel in 1887 [24] as a *complex shift*. Appel noticed that Coulomb solution

$$\phi(\vec{x}) = e/r = e/\sqrt{x^2+y^2+z^2} \tag{13}$$

is invariant under the shift $\vec{x} \to \vec{x} + \vec{a}$, and considered complex shift of the origin, $(x_0, y_0, z_0) = (0,0,0)$ along z-axis. $(x_0, y_0, z_0) = (0,0,-ia)$. On the real slice he obtained the complex potential

$$\phi_a(\vec{x}) = Re\ e/\tilde{r}, \tag{14}$$

with complex radial coordinate $\tilde{r} = r + ia\cos\theta$. It was shown in [2, 32, 34] that potential (14) corresponds exactly to KN electromagnetic field, and the exact KN solution may be described as a field generated by a **complex source propagating along complex world-line**

$$x_L^\mu(\tau_L) = x_0^\mu(0) + u^\mu \tau_L + \frac{ia}{2}\{k_L^\mu - k_R^\mu\}, \tag{15}$$

where $u^\mu = (1,0,0,1)$, $\quad k_R = (1,0,0,-1)$, $\quad k_L = (1,0,0,1)$. Index L labels it as a Left structure, and we should add a complex conjugate Right structure

$$x_R^\mu(\tau_R) = x_0^\mu(0) + u^\mu \tau_R - \frac{ia}{2}\{k_L^\mu - k_R^\mu\}. \tag{16}$$

Therefore, from complex point of view the Kerr and Schwarzschild geometries are equivalent and differ only by their *real slice*, which for the Kerr solution goes aside of its center.

Complex shift turns the Schwarzschild radial directions $\vec{n} = \vec{r}/|r|$ into twisted directions of the Kerr congruence, Fig.1.

4 Complex Kerr's string

It was obtained [23, 26] that the complex world line $x_0^\mu(\tau)$, parametrized by complex time $\tau = t + i\sigma$, represents really a two-dimensional surface which takes an intermediate position between particle and string. The corresponding "hyperbolic string" equation [26], $\partial_\tau \partial_{\bar\tau} x_0(t,\sigma) = 0$, yields the general solution

$$x_0(t,\sigma) = x_L(\tau) + x_R(\bar\tau) \tag{17}$$

as sum of the analytic and anti-analytic modes $x_L(\tau), \quad x_R(\bar\tau)$, which are not necessarily complex conjugate. For each real point x^μ, the parameters τ and $\bar\tau$ should be determined by a complex retarded-time construction. Complex source of the KN solution corresponds to two *straight* complex conjugate world-lines,(15),(16). Contrary to the real case, the complex retarded-advanced times $\tau^\mp = t \mp \tilde{r}$ may be determined by two different (Left or Right) complex null planes, which are generators of the complex light cone. It yields four different roots for the Left and Right complex structures [32, 34]

$$\tau_L^\mp = t \mp (r_L + ia\cos\theta_L) \tag{18}$$
$$\tau_R^\mp = t \mp (r_R + ia\cos\theta_R). \tag{19}$$

The real slice condition determines relation $\sigma = a\cos\theta$ with null directions of the Kerr congruence $\theta \in [0,\pi]$, which puts restriction $\sigma \in [-a,a]$ indicating that *the complex string is open*, and its endpoints $\sigma = \pm a$ may be associated with the Chan-Paton charges of a quark-antiquark pair. In the real slice, the complex endpoints of the string are mapped to the north and south twistor null lines, $\theta = 0, \pi$, see Fig.3.

Orientifold. The complex open string boundary conditions [23] require the *worldsheet orientifold* structure [27, 35, 36, 37, 38] which turns the open string in a closed but folded one. The world-sheet parity transformation $\Omega : \sigma \to -\sigma$ reverses orientation of the world sheet, and covers it second time in mirror direction. Simultaneously, the Left and Right modes are exchanged. [3] The projection Ω is combined with space reflection $R : r \to -r$, resulting in $R\Omega : \tilde{r} \to -\tilde{r}$, which relates the retarded and advanced folds

$$R\Omega : \tau^+ \to \tau^-, \tag{20}$$

preserving analyticity of the world-sheet. The string modes $x_L(\tau), \quad x_R(\bar\tau)$, are extended

[3] Two oriented copies of the interval $\Sigma = [-a,a]$, $\Sigma^+ = [-a,a]$, and $\Sigma^- = [-a,a]$ are joined, forming a circle $S^1 = \Sigma^+ \bigcup \Sigma^-$, parametrized by θ, and map $\theta \to \sigma = a\cos\theta$ covers the world-sheet twice.

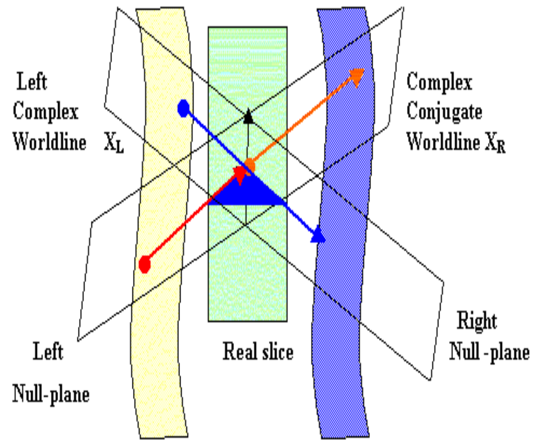

Figure 2: The complex conjugate Left and Right null planes generate the Left and Right retarded and advanced roots.

on the second half-cycle by the well known extrapolation, [27, 38]

$$x_L(\tau^+) = x_R(\tau^-); \quad x_R(\tau^+) = x_L(\tau^-), \tag{21}$$

which forms the folded string, in with the retarded and advanced modes are exchanged every half-cycle.

The real KN solution is generated by the straight complex world line (CWL) (15) and by its conjugate Right counterpart (16). By excitations of the complex string, the orientifold condition (21) becomes inconsistent with the complex conjugation of the string ends, and *the world lines $x_L(\tau)$, and $x_R(\bar\tau)$ should represent independent complex sources.* The projection $\mathcal{T} = R\Omega$ sets parity between the positive Kerr sheet determined by the Right retarded time and the negative sheet of the the Left advanced time. It allows one to escape the anti-analytical Right complex structure, replacing it by the Left advanced one, and the problem is reduced to self-interaction of the retarded and advanced sources determined by the time parameters $\tau^\pm$. For any non-trivial (not straight) CWL, the Kerr theorem will generate different congruences for τ^+, and τ^-. Each of these sources produces a twosheeted Kerr-Schild geometry, and the formal description of the resulting four-folded congruence should be based on the multi-particle Kerr-Schild solutions, [33].[4] The corre-

[4]Physical motivation of such a splitting of the sources is discussed in seminal paper by De Witt and Breme, [39], where authors introduce the similar 'bi-tensor' fields, which are predecessors of the two-point Green's functions and Feynman propagator. The problem of physical interpretation goes beyond frame of this paper, and will be considered elsewhere.

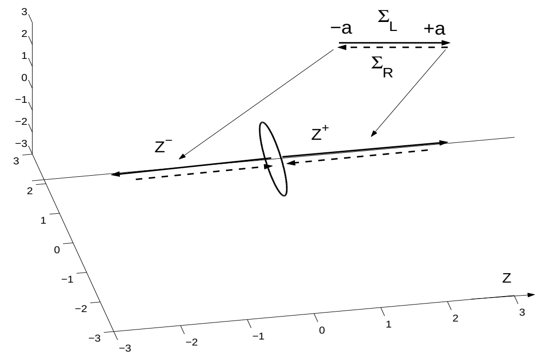

Figure 3: Ends of the open complex string, associated with quantum numbers of quark-antiquark pair, are mapped onto the real half-infinite z^+, z^- axial strings. Dotted lines indicate orientifold projection.

sponding two-particle generating function of the Kerr theorem will be

$$F_2(T^A) = F_L(T^A)F_R(T^A), \tag{22}$$

where F_L and F_R are determined by $x_L(\tau^+)$, and $x_L(\tau^-)$. The both factors are quadratic in T^A. The corresponding equation

$$F_2(T^A) = 0 \tag{23}$$

describes *a quartic in* CP^3 which is the well known Calabi-Yau two-fold, [27, 38]. We arrive at the result that excitations of the Kerr complex string generate a Calabi-Yau two-fold, or K3 surface, on the projective twistor space CP^3.

5 Outlook.

One sees that the Kerr-Schild geometry displays striking parallelism with basic structures of superstring theory. However, our principal result in this paper is the presence of inherent Calabi-Yau twofold in the complex twistorial structure of the Kerr geometry. In the recent paper [18] we argued that it is not accidental, because gravity is a fundamental part of the superstring theory. However the Kerr-Schild gravity, being based on twistor theory, displays also some inherent relationships with superstring theory.

In many respects the Kerr-Schild gravity resembles the twistor-string theory, [40, 41, 21], which is also four-dimensional, based on twistors and related with experimental particle physics. On the other hand, the complex Kerr string has mach in common with the N=2 superstring [26, 38, 43]. It is also related with twistors and has the complex critical

dimension two which corresponds to four real dimensions and indicated that N=2 superstring may lead to four-dimensions. However, signature of the N=2 string may only be (2,2) or (4,0), which caused the obstacles for embedding of this string in the space-times with minkowskian signature. Up to our knowledge, this trouble was not resolved so far, and the initially enormous interest to N=2 string seems to be dampened. Meanwhile, embedding of the N=2 string in the complexified Kerr geometry is almost trivial task. It hints that stringlike structures of the real and complex Kerr geometry are not simply analogues, but reflect the underlying dynamics of the N=2 superstring theory,

In the same time, along with wonderful parallelism, the stringy system of the four-dimensional KN geometry displays very essential peculiarities. It is applicable to particle physics, which have great spin/mass ratio, for which the black hole horizons disappear, opening the naked Kerr ring, resulting in the over-rotating BH geometry.

- The supplementary Kaluza-Klein space is absent, and the role of compactification circle is played by the Kerr singular ring with traveling waves, which realizes a "compactification without compactification", [18].
- The lightlike twistorial rays are tangent to the Kerr singular ring, indicating that the Kerr ring is the lightlike string, and it may play the role of DLCQ circle in M(atrix) theory, [42].
- Consistency of the KN solution with gravitational background of the electron, [8, 9, 13, 18], shows that the 4d Kerr characteristic length of the Kerr ring, $a = J/m$, corresponds to the Compton scale of spinning particles.

The considered stringy structures of the real and complex Kerr geometry set a parallelism between the 4d Kerr geometry and superstring theory, indicating that complexification of the Kerr geometry may serve an alternative to traditional compactification of higher dimensions.

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References

[1] R.P. Kerr, Phys. Rev. Lett. **11**, 237 (1963).

[2] A.Ya. Burinskii, Sov. Phys. JETP **39**, 193 (1974).

[3] D.D. Ivanenko and A.Ya. Burinskii, Izv. Vuz. Fiz. **5** 135 (1975).

[4] A. Burinskii, Phys. Rev. D **52**, 5826 (1995), [arXiv:hep-th/9504139].

[5] A. Sen, "Rotating charged black hole solution in heterotic string theory", Phys. Rev. Lett. **69**, 1006 (1992).

[6] A. Burinskii, *First Award Essay of GRF 2009,* Gen. Rel. Grav. **41**, 2281 (2009), [arXiv:0903.3162].

[7] H. Keres, ZhETP **52** iss.3 (1967) 768 (in Russian).

[8] W. Israel, Phys. Rev. D **2**, 641 (1970).

[9] C.A. López, Phys. Rev. D **30**, 313 (1984).

[10] A. Burinskii, Grav. Cosmol. **8**, 261 (2002), [arXiv:hep-th/0110011].
A. Burinskii, J. Phys. A: Math. Gen. **39**, 6209 (2006).

[11] A. Burinskii, J. Phys. A: Math. Theor. **43** 392001 (2010), [arXiv: 1003.2928].

[12] E. T. Newman et al. J. Math. Phys., **6**, 918 (1965).

[13] B. Carter, Phys. Rev. **174**, 1559 (1968).

[14] G. C. Debney, R. P. Kerr and A. Schild, J. Math. Phys. **10**, 1842 (1969).

[15] Th.M. Nieuwenhuizen, In: "Beyond the Quantum", eds. Th.M. Nieuwenhuizen et al., (World Scientific, Singapure) 2007, pp.332-342.

[16] A. Burinskii, Grav. Cosmol. **14**, 109 (2008), [arXiv:hep-th/0507109].

[17] A. Burinskii, Grav. Cosmol.**10**, 50 (2004).

[18] A. Burinskii, J. of Phys.: Conf. Series, (2012) (to appear), [arXiv:1109.3872].
A. Burinskii, J, of Physics: Conference Series, **343** 012019 (2012), [arXiv:1112.0225].

[19] A. Burinskii, Proc. of the Conf. "Beyond the Quantum", eds. Th.M. Nieuwenhuizen et al., (World Scientific, Singapure) 2007, p.319, [gr-qc/0606035].

[20] A. Burinskii, Theor. Math.Phys. **163**(3), 782 (2010), [arXiv:0911.5499]
A. Burinskii, J. Phys.: Conf. Ser. **222** 012044 (2010), [arXiv:1001.0332].

[21] A. Burinskii, Phys. Rev. D **70**, 086006 (2004), [arXiv:hep-th/0406063].

[22] I. Dymnikova, Phys. Lett. B **639**, 368 (2006).

[23] A.Ya. Burinskii, *String-like Structures in Complex Kerr Geometry,* In: “Relativity Today”, Edited by R.P.Kerr and Z.Perjés, Akadémiai Kiadó, Budapest, 1994, p.149, [arXiv:gr-qc/9303003].

A. Burinskii, Phys. Lett. A **185**, 441 (1994).

A.Ya. Burinskii, Phys. Rev.**D 57** 2392 (1998).

[24] E.T. Whittacker and G.N. Watson,“A Course of Modern Analysis”, Cambrige Univ. Press London/New York,p.400, 1969.

[25] E. T. Newman, J. Math. Phys. **14**, 102 (1973).

R.W. Lind, E.T. Newman, J. Math. Phys. **15** 1103 (1974).

[26] H. Ooguri and C. Vafa. Nucl. Phys. **B361** 469 (1991); ibid **B.367** 83 (1991).

[27] K. Becker, M. Becker and J. Schwarz, ‘String Theory and M-Theory - A Modern Introduction”, Cambridge University Press, 2007.

[28] L.M. Dyson, L. Järv and C. V. Johnson, *Oblate, Toroidal, and Other Shapes for the Enhancon, JHEP* **0205** (2002) 019 [arXiv: hep-th/ 0112132],

[29] D.Kramer, H.Stephani, E. Herlt, M.MacCallum, “Exact Solutions of Einstein’s Field Equations”, Cambridge Univ. Press, Cambridge 1980.

[30] R. Penrose, J. Math. Phys. **8** 345 (1967).

[31] R. Penrose, W. Rindler, “Spinors and space-time, ” v.2. Cambridge Univ. Press, England, 1986.

[32] A. Burinskii and R.P. Kerr, *Nonstationary Kerr Congruences*, [e-print gr-qc/9501012].

[33] A. Burinskii, Grav. Cosmol, **11**, 301 (2005), [hep-th/0506006].

A. Burinskii, Grav. Cosmol. **12**, 119 (2006).

A. Burinskii, Int.J. Geom.Meth.Mod.Phys. **4**, n.3 437 (2007), [hep-th/0510246].

[34] A. Burinskii, Phys. Rev. **D 67**, 124024 (2003).

[35] Dixon L., Harvey J.A., Vafa C., Witten E. Nucl. Phys. **B 261**, 678 (1985); ibid. Nucl. Phys. **B 274** 285 (1986).

[36] Hamidi S., Vafa C. Nucl. Phys. **B 279**, 465 (1987).

[37] P. Horava, Nucl. Phys. **B 327** 461 (1989).

[38] M.B. Green, J. Schwarz, and E. Witten, “ Superstring Theory”, v.I, Cambridge Univ. Press, 1987.

[39] B. S. De Witt and R. W. Breme, Ann. of Phys. **9**, 220 (1960).

[40] V.P. Nair, Phys.Lett. **B 214** (1988) 215.

[41] E. Witten, Comm. Math. Phys. **252**, 189 (2004), [hep-th/0312171].

[42] L. Susskind, *Another Conjecture about M(atrix) Theory,* hep-th/9704080

[43] A. DAdda and F. Lizzi, *Space Dimensions from Supersymmetry for the $N = 2$ Spinning String: A Four Dimensional Model,* Phys. Lett. 191B (1987) 85.